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Modernising Cash Flow Matching

Tetris, the addictive computer game where falling tetrominoes must be lined up to fill the available space, was invented in 1984. Despite there being many more sophisticated games available today, it remains popular because, like all good games, Tetris has a clear objective with simple rules and is difficult to master. In 2003 mathematicians from MIT revealed that the reason Tetris is so challenging is that it belongs to a class of problems beloved by cryptologists. Cryptologists study codes and a good code is one that is easy to unlock if you have the key, but where the task of finding that key is hard. For a cryptologist a hard task is one that takes a long time to complete.

Cash flow matching has features in common with Tetris: it is one of the oldest techniques for matching assets to liabilities; the objective of cash flow matching is clear, but finding a perfect solution is very difficult; and despite more sophisticated approaches being available it remains popular.

Cash flow matching involves putting together a portfolio of bonds with cash flow receipts that match the timing and scale of expected cash flow payments. Figure 1 is an example of the distribution of cash flows that a matching portfolio would seek to replicate. The graph shows the proportion of cash flows payable by an annuity provider within a certain time period: in this example the final cash flow is in 90 years. A cash flow matching portfolio would consist of bonds that together have cash flows distributed similarly. While approximate matches are achievable, exact matches are hard to find.

In Figure 2 we have added the cash flows from a portfolio that approximately matches those of Figure 1. The shaded areas represent the gap between the asset and liability cash flows. The small blue shaded area around 10 years illustrates that in this section of the curve the asset cash flows exceed those of the liabilities – the asset curve bulges through the liability curve. At tenors greater than 20 years the situation reverses and the asset curve fails to reach the liability curve. In this example, the value of the assets is equal to that of the liabilities.

This is a relatively good match, but it is not perfect and achieving perfection is hard in the sense meant by cryptologists. The liability curve consists of about one thousand cash flows and to match each of those exactly would be a laborious task, even if practical. Like Tetris, we have to find bonds with the right shape of cash flows to fill the gaps.

Figure 1: Cumulative liability cash flows by tenor

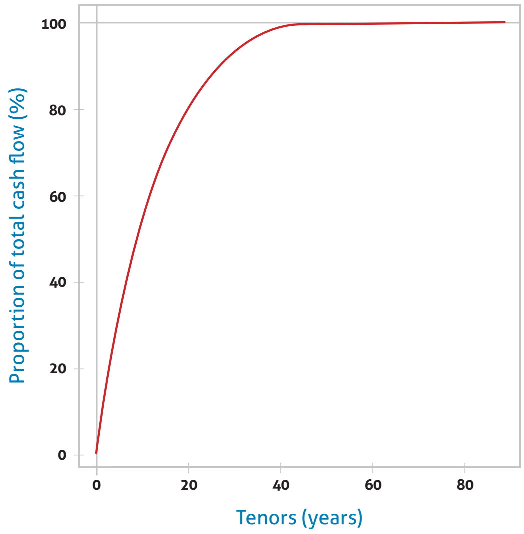
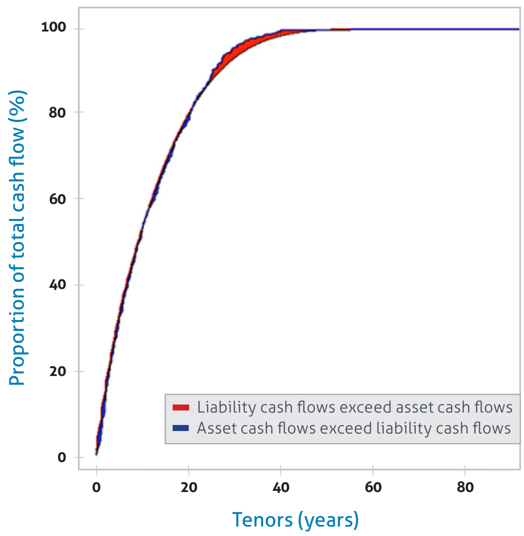


Figure 2: Cumulative asset and liability cash flows by tenor



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How Good a Match is Good Enough?

The question of how good a match is good enough has exercised bond managers and actuaries for a very long time. Writing in 1952 a British actuary Frank Redington[1] suggested that rather than seek to achieve a perfect cash flow match a good way to control the economic exposures of the portfolio was to require the blue areas and red areas to be equal in extent. He termed this approach "immunisation" and showed that the difference in the size of the blue and red areas was the net duration of the portfolio. His argument was instrumental in duration being adopted as a control measure in fixed income portfolio management. In the example shown in Figure 2, the timing of the cash flows is such that the red area is equal to the blue areas and therefore we say that the net assets of the annuity business are duration neutral: the net assets will not change for a small increase or decrease in interest rates.

Redington's proposal suggests not only a way to judge the goodness of our fit, but also a way to improve it. He summarised the thousand plus liability cash flows with a single number, which is an audacious simplification. For Star Trek fans, Redington's approach to cash flow matching was like James T Kirk's approach to the Kobayashi Maru scenario. Like Kirk, Redington opted not to play the game as presented but to write his own rules. If he were to match the thousand plus cash flows of the liabilities, then he would need an equivalent number of zero coupon bonds. His approach would allow him to match his approximate curve with a single instrument.

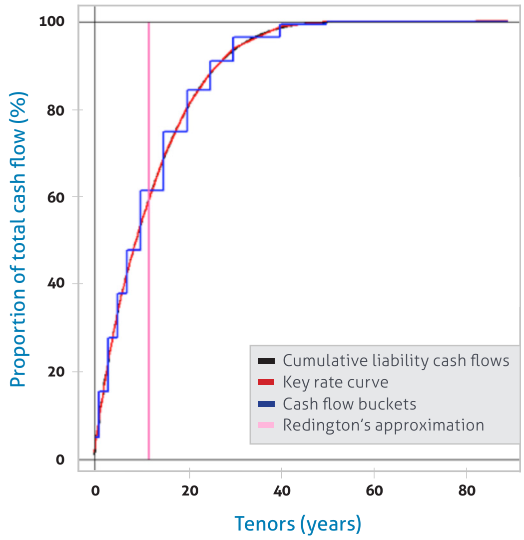
Redington borrowed a technique often used by mathematicians: if we cannot solve a problem exactly, then can we find a similar but simpler problem that we can solve? Rather than seek to achieve a perfect match to the liabilities, Redington's approach is to match an approximation of the liability curve. This approximation is shown in Figure 3 and the startling simplicity of his technique is evident.

Redington provided a practical way to approximate the liabilities for a time before we had computers. If we could precisely describe every bond with a single number, then we could perfectly match Redington's curve. Unfortunately, the only bonds that can be so described are zero coupon bonds, which are as rare as hen's teeth. Today, common practice is to approximate the curve by a stepped function, like that in Figure 3, and if we could precisely describe each bond in a similar fashion using cash flow buckets, then we could perfectly match the stepped curve. This description is a better reflection of the reality of a bond's cash flows, but because we generally fix the location of the steps we introduce a further complication. As time passes cash flows fall from one step to the next, moving suddenly from one bucket to one spanning shorter tenors. This causes measures of the quality of the fit to jump around. For this reason, at Cameron Hume we have moved to using so-called key rate curves. These curves both describe the liability curve better, as Figure 3 shows, and do not suffer from sudden transitions. In fact, approximations are nearly indistinguishable from the actual cash flows.

The strategy developed first by Redington and elaborated upon since offers a simpler approach to asset liability management than cash flow matching by reducing the number of features that we need to match. Redington's strategy had a single feature, the cash flow buckets in Figure 3 have 12 and the key rates 13 features. This is vastly less than the thousand or so cash flows of the full description and is easily and readily solvable with a simple portfolio optimiser.

"The strategy developed first by Redington and elaborated upon since offers a simpler approach to asset liability management"

Figure 3: Cumulative liability cash flows by tenor



[1] "Review of the principles of life-office valuation", Institute of Actuaries, Redington 1952.

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Cash Flow Matching for Today

In a real world asset liability management problem the quality of the cash flow match is only one of many criteria that an asset liability management policy must satisfy. The task is complicated by the need to meet diversity and credit quality constraints, to achieve an investment return target subject to a limit on capital and to take into account any under or over funding of the business. It is in these circumstances that the simplicity of the Redington approach becomes a disadvantage. We can illustrate this in part with a simple example. In Figure 4 we asked the optimiser to choose bonds that maximise the expected return of the assets while matching the characteristics of our various approximations to the liability curve, subject to mild additional constraints and that no bond may be more than 20% of the assets. In Figure 5 we repeat the exercise, but this time no bond may represent more than 1% of the assets. Adding this single constraint causes the cash flow fit from Redington's approach to change, but has less effect on the other two approaches. We used this Redington solution to create Figure 2 and the key rate curve in Figure 5 shows a near perfect fit.

Cash flow matching puts a disproportionate focus on the fine detail of the interest rate exposures and crowds out consideration of other factors. Redington's approach is at the opposite extreme and imposes too little control on the interest rate exposures. The more modern approaches seek an optimal intermediate way that combines sufficient control, stable solutions and intuitively simple means of monitoring the net exposures of the assets and liabilities. The key rate approximation solves the cash flow matching problem and more elaborate approximations allow greater control and better oversight of other exposures.

The ease with which players can learn the rules of Tetris is one explanation for its continuing popularity, but another is that it is simply challenging. Cash flow matching is also simple to understand and it is challenging to implement, but what is addictive in Tetris is simply a hindrance in asset liability management. It is time to trade in cash flow matching for more modern approaches that are designed for today's technology not that of 1950s Britain.

Figure 4: Cumulative asset and liability cash flows by tenor

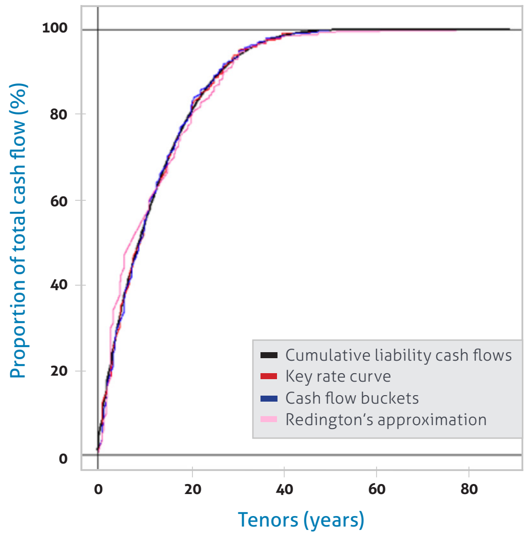
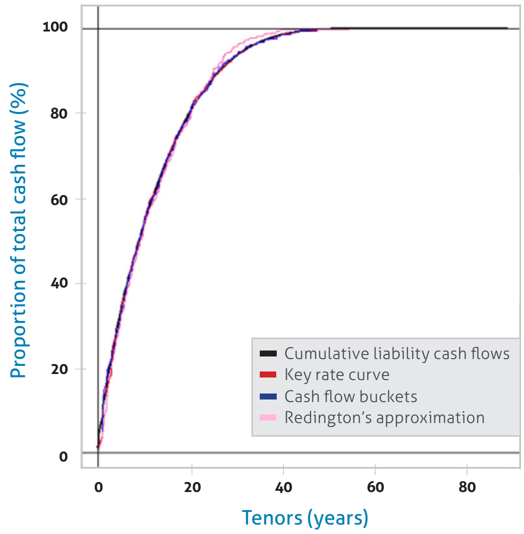


Figure 5: Cumulative asset and liability cash flows by tenor



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